This question paper consists of 9 pages and 1 information sheet.
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.

2. Answer ALL the questions.

3. Number the answers correctly according to the numbering system used in this question paper.

4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining the answers.

5. Answers only will NOT necessarily be awarded full marks.

6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

7. If necessary, round off answers to TWO decimal places, unless stated otherwise.

8. Diagrams are NOT necessarily drawn to scale.

9. An information sheet with formulae is included at the end of the question paper.

10. Write neatly and legibly.
QUESTION 1

1.1 Solve for $x$:

1.1.1 $2x(5x - 3) = 0$ (2)

1.1.2 $-x^2 + 4 = 5x$ (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $\sqrt{x - 6} - 2 = \frac{15}{\sqrt{x - 6}}$ (5)

1.1.4 $(x^2 + 2)(x - 3) < 0$ (2)

1.2 Solve for $x$ and $y$ simultaneously:

$x + 2y = 3$ and $3x^2 + 4xy + 9y^2 - 16 = 0$ (6)

1.3 Determine the sum of the digits of: $2^{2022} \cdot 5^{2018}$ (4) [23]

QUESTION 2

2.1 If $x + 1; x - 1; 2x - 5$ are the first 3 terms of a geometric series, calculate:

2.1.1 The value(s) of $x$ (5)

2.1.2 For which value of $x$ in QUESTION 2.1.1 will the series converge? (4)

2.1.3 The sum to infinite if $x = 3$ (2)

2.2 If you had a choice between receiving 3 million rand on your 21st birthday, or rather R1 on your 21st birthday, R2 on the day after your birthday, R4 on the 2nd day after your birthday and so forth, until the end of the month for 30 days, what will you choose? Show all calculations to motivate your answer. (3) [14]
QUESTION 3

The seats in an open-air theatre are numbered as in the sketch below.

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<table>
<thead>
<tr>
<th>Row 1</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row 2</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Row 3</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>Row 4</td>
<td>22</td>
<td>23</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Row 5</td>
<td>33</td>
<td>34</td>
<td>35</td>
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<td>37</td>
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<td>42</td>
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<tr>
<td></td>
<td>43</td>
<td>44</td>
<td>45</td>
</tr>
</tbody>
</table>
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3.1 If 1; 6; 13; 22; 33 forms a sequence of seat numbers of the first seat in each row, determine the $n^{th}$ term of this row. (4)

3.2 Hence, find the seat number of the $7^{th}$ seat in row 15. (2)

3.3 How many chairs are in the $25^{th}$ row? (3)

3.4 The theatre manager bought 2000 chairs. Use a relevant formula to determine how many complete rows can be placed. (5)

[14]

QUESTION 4

Given: $f(x) = \frac{a}{x-p} + q$.

- The point B(−1; 0) is an x-intercept of $f$.
- The domain of $f$ is real numbers, but $x \neq 2$.
- The range of $f$ is real numbers, but $y \neq 3$.
- $f$ is a decreasing function.

4.1 Determine the equation of $f$. (3)

4.2 Determine the coordinates of the y-intercept of $f$. (2)

4.3 Sketch the graph of $f$ in your ANSWER BOOK, clearly showing the asymptotes and the intercepts with the axes. (3)

[8]
QUESTION 5

The graphs of $f(x) = a(x + p)^2 + q$ and $g(x) = \frac{2}{x + 1} - 3$ are sketched below. P is the $y$-intercept of $f$ and $g$. The horizontal asymptote of $g$ is also a tangent to $f$ at the turning point of $f$.

5.1 Write down the equation of the vertical asymptote of $g$. (1)

5.2 Determine the coordinates of $P$. (2)

5.3 Determine the equation of $f$. (3)

5.4 One of the axes of symmetry of $g$ is a decreasing function. Write down the equation of this axis of symmetry, $h(x)$. (2)

5.5 For which values of $k$ will $g(x) = h(x) + k$ have TWO real roots that are of opposite signs? (2)

5.6 Give the domain of $m(x)$ if $m(x) = g(2x) + 5$. (3)

[13]
QUESTION 6

The diagram below shows the curves of \( f(x) = \frac{1}{4} x^2 \), where \( x \geq -2 \) and \( g(x) = a^x \), where \( a > 0 \). The point \( A(-1 ; 3) \) lies on the graph of \( g \).

![Graph of f and g with point A(-1;3)]

6.1 Show that \( g(x) = \left( \frac{1}{3} \right)^x \). \( \text{(1)} \)

6.2 For which value(s) of \( x \) is the graph of \( f \) strictly decreasing? \( \text{(2)} \)

6.3 Determine the inverse of \( f \) in the form \( y = \ldots \). \( \text{(2)} \)

6.4 Sketch the graph of \( f^{-1} \) in your ANSWER BOOK. \( \text{(2)} \)

6.5 Write down the range of \( f^{-1} \). \( \text{(2)} \)

6.6 Determine the inverse of \( g \) in the form \( y = \ldots \). \( \text{(2)} \)

6.7 For which values of \( x \) will \( g^{-1}(x) \geq -1 \)? \( \text{(2)} \)
QUESTION 7

7.1 William receives an amount of R2 000 000 upon his retirement. He invests this amount immediately at an interest rate of 9.3% per annum, compounded monthly.

He needs an amount of R35 000 per month to maintain his current lifestyle. He plans to withdraw the first amount by the end of the first month.

For how many months will he be able to live from his investment? (5)

7.2 John has just bought a new truck for R1 200 000. He has decided to replace the truck in 6 years’ time, when the trade-in value of the truck would be R400 000. The replacement cost of the truck is expected to increase by 7.5% per annum.

7.2.1 Take into account the trade-in value of his old truck and calculate how much cash John will need in 6 years’ time to buy a new truck. (3)

7.2.2 John starts a sinking fund to provide for a new truck. Determine the quarterly deposit if his first deposit is made 6 months after he bought his truck and his last deposit 6 months before the end of the 6 years. The interest rate for the money in the sinking fund is 11% per annum, compounded quarterly. (5)

7.2.3 Suppose that 2 years after the purchase of his truck and every year thereafter for the next 3 years, John wants to withdraw R8 000 from his account, to pay for services for his truck. If he makes 4 such withdrawals, what will the new quarterly deposit be during this time period? (4)

QUESTION 8

8.1 If \( f(x) = 2x^2 - 5x + 3 \), determine \( f'(x) \) from first principles. (5)

8.2 Determine \( \frac{dy}{dx} \) if \( y = \frac{2x^2}{3\sqrt{x}} - \frac{2x^3 + 1}{x^3} \). (5)
QUESTION 9

9.1  Given: \( f(x) = -2x^3 + 5x^2 + 4x - 3 \)

9.1.1 Calculate the coordinates of the \( x \)-intercepts of \( f \) if \( f(3) = 0 \). Show all calculations. \((4)\)

9.1.2 Calculate the \( x \)-values of the stationary points of \( f \). \((4)\)

9.1.3 For which values of \( x \) is \( f \) concave up? \((2)\)

9.2  The function \( g \), defined by \( g(x) = ax^3 + bx^2 + cx + d \) has the following properties:

- \( g(-2) = g(4) = 0 \)
- The graph of \( g'(x) \) is concave up.
- The graph of \( g''(x) \) has \( x \)-intercepts at \( x = 0 \) and \( x = 4 \) and a turning point at \( x = 2 \).

9.2.1 Use this information to draw a neat sketch graph of \( g \) without actually solving for \( a \), \( b \), \( c \) and \( d \). Clearly show all \( x \)-intercepts, \( x \)-values of the turning points and \( x \)-value of the point of inflection on your sketch. \((4)\)

9.2.2 For which values of \( x \) will \( g(x) \cdot g''(x) > 0 \)? \((3)\)  \((17)\)

QUESTION 10

A shopkeeper finds that the number of people visiting his shop at any moment during the 10 hours that the shop is open, is represented by:

\[ N(t) = t^3 - 12t^2 + 36t + 8 \]

where \( N(t) \) is the number of people in the shop, \( t \) hours after the shop opened.

10.1 How many people are in the shop when the shop opens? \((1)\)

10.2 At what stage is the number of people in the shop increasing? \((5)\)

10.3 At which stage is it the best time for the shopkeeper to take a break and leave his assistant alone in the shop? \((1)\)  \((7)\)
QUESTION 11

11.1 N and M are two events. \(P(N) = 0.3; \ P(M) = 0.4\) and \(P(M \ or \ N) = 0.6\).

11.1.1 Sketch a Venn-diagram to represent the events. \(\text{ (5)}\)

11.1.2 Are the events N and M independent? Motivate your answer by showing all relevant calculations. \(\text{ (5)}\)

11.2 A five-digit code is created by using digits 0 to 9. Digits may not be repeated. How many different codes are possible if the code must be a multiple of 5 and the code must start with an 8? \(\text{ (4)}\) \[14\]

TOTAL: 150
INFORMATION SHEET: MATHEMATICS

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ A = P(1 + ni) \quad A = P(1 - ni) \quad A = P(1 - i)^n \quad A = P(1 + i)^n \]

\[ T_n = a + (n - 1)d \quad S_n = \frac{n}{2} [2a + (n - 1)d] \]

\[ T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1 \quad S_n = \frac{a}{1 - r}; \quad -1 < r < 1 \]

\[ F = \frac{\sqrt{(1 + i)^n - 1}}{i} \quad P = \frac{\sqrt{1 - (1 + i)^{-n}}}{i} \]

\[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \]

\[ y = mx + c \quad y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan \theta \]

\[ (x - a)^2 + (y - b)^2 = r^2 \]

In \( \triangle ABC \):

\[ \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \]

\[ \text{area } \triangle ABC = \frac{1}{2} ab \sin C \]

\[ \sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta \quad \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \]

\[ \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \]

\[ \cos 2\alpha = \begin{cases} 
\cos^2 \alpha - \sin^2 \alpha \\
1 - 2 \sin^2 \alpha \\
2 \cos^2 \alpha - 1 
\end{cases} \quad \sin 2\alpha = 2 \sin \alpha \cos \alpha \]

\[ \bar{x} = \frac{\sum x}{n} \quad \sigma^2 = \frac{\sum (x_i - \bar{x})^2}{n} \]

\[ P(A) = \frac{n(A)}{n(S)} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

\[ \hat{y} = a + bx \]

\[ b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2} \]

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